Network Coding in Future Mobile Networks (IMT-A & Beyond)

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Outline

- General Introductions
- Multi-user cooperative networks
- Multi-user multi-relay networks
- DMT analysis
- Network coding for backhaul-supported networks
- Conclusions

Background

- IMT-Advanced & Beyond: (International Mobile Telecommunications), simply 4G, defined by ITU.

- Objective: 100Mbps -> 1Gbps. Do we need so much?

  --- The first commerical networks were deployed at Stockholm and Olso, December 2009. One typical application: HDTV.

- Many candidate standards: LTE, WiMAX, UMB etc. LTE (by 3GPP) is one of the most important.

Background (Cont'd)

- Research Projects (quite many) for IMT-A & B.
- In Europe, WINNER (I, II, +): Wireless Initiative New Radio, about 40 partners world-widely, coordinated by NSN, including major players (Ericsson, Qualcomm, Moto, ...).
- WINNER+: Innovations are expected in radio-resource management, spectrum sharing and its flexible usage, exploitation of peer-to-peer links between user terminals, incorporation of network coding and advanced antenna concepts.

Background (Cont'd)

- Main technologies:
  1. Optimized Resource Management: Radio resource.
  2. Strong error coding: Turbo-like codes.
  3. Carrier Aggregation
  4. Spectrum Sensing
  5. Multi-user MIMO
  6. Coordinated MultiPoint access (CoMP)
  7. Relaying
  8. Network Coding
  9. Device-to-device communications

Part I: Multi-user Cooperation with Linear Network Coding

Based on:

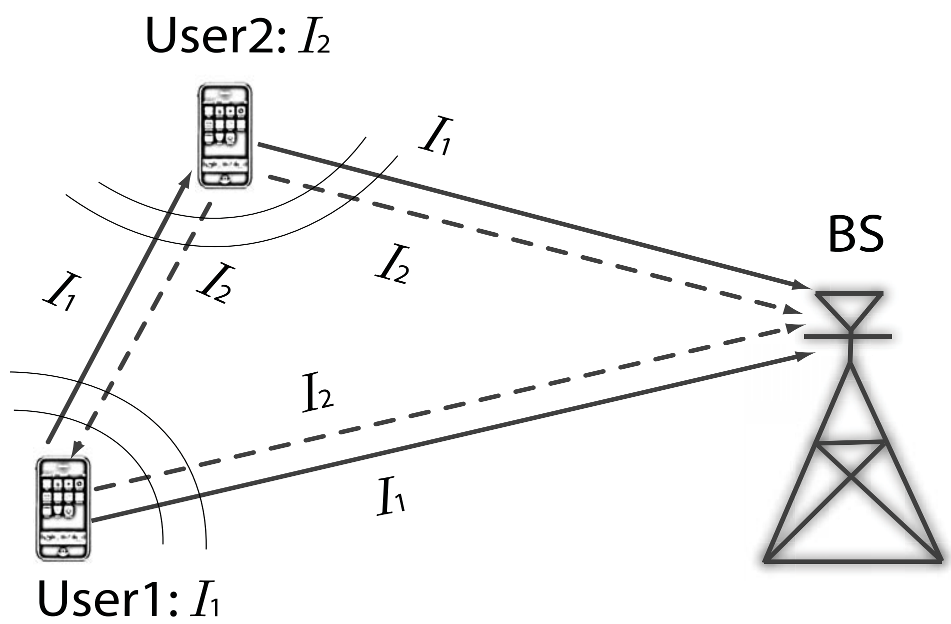
M. Xiao and M. Skoglund, “Multiple-user cooperative communications based on linear network coding”, IEEE Transactions on Communications, vol. 58, no. 11, November 2010.

Outline of Part I

- 2-user cooperative networks
- Multiple-user cooperative networks
- Conclusions

Background

- Cooperative communications has attracted lots of research efforts in recent years. Partners help each other to relay information.



User1 and User2 help each other by relaying overheared messages. Diversity order 2 for each user.

- Typically, cooperation communications separates local and relaying message: routing scheme.

Background (cont'd)

- Network coding allows information combining in the intermediate nodes, thus it is a natural application in cooperative communications, gains in energy efficiency.

- Network coding schemes in [1-3] are not well designed. Thus, gain is limited, especially for multi-user networks.

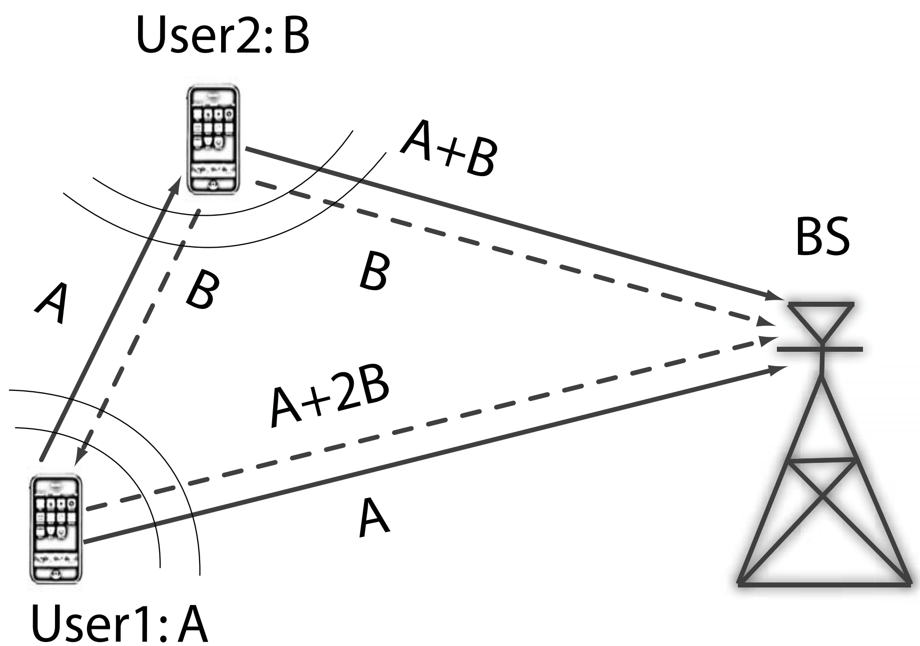
[1] L. Xiao, T. Fuja, J. Kliewer, and D. Costello, "A network coding approach to cooperative diversity," *IEEE Trans. on Inform. Theory*, vol. 53, no. 10, pp. 3714-3722, Oct. 2007.

[2] Y. Chen, S. Kishore and J. Li, "Wireless diversity through network coding," in *Proc. IEEE WCNC 2006*, pp. 1681–1686

[3] P. Larsson, N. Johansson, and K. E. Sunell, "Coded bi-directed relaying," In *IEEE Vehicular Technology*, pp. 851-855, May 2006.

2-user networks

- System description: MDS network codes are used in relaying nodes.



4 received blocks: A, B, A+B, A+2B. Any 2 can rebuild A and B.

Challenging: (1) Topology is time-varying. (2) Prefer to fixed codes.

- Achieved the min-cut (2) for dynamic networks. Each block has independent fading (block fading). High diversity is thus achieved.

2-user networks (cont'd)

- Performance analysis:

Assuming block fading channels:

$$Y = aX + N;$$

X, Y are transmitted and received codewords. a fading coeff.

$$MI = \log(1+ |a|^2 SNR).$$

Rayleigh fading, an outage occurs in a channel in probability

$$P_e \approx (2^R-1)/SNR.$$

Only 3 or more blocks in outage cause an outage in network

With the outage in inter-user channels, we eventually can get the overall outage probability as

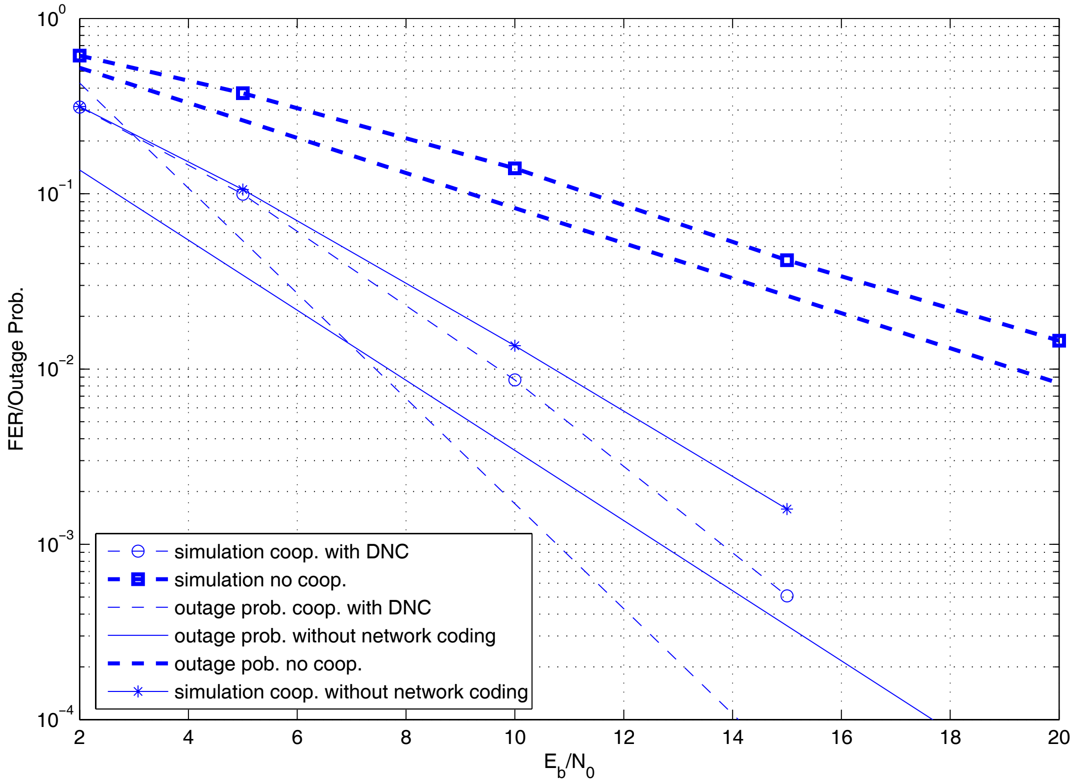
$$P_{1,o} \approx 3.5 \, P_e^3.$$

Diversity order 3 is achieved, higher than previous schemes

2-user networks (cont'd)

- Numerical results

Reciprocal interuser channel. (200, 400, 3) LDPC codes as channel codes.



Multi-user networks

- Protocol:

M-user networks, each user transmitts M codewords: one for direct trans and M-1 for relaying, all by orthogonal channels.

Network codes design, Diversity network codes (DNC): the BS can recover I_j if it successfully received any $|D_j|$ codewords from outputs of D_j (the set of users relaying I_j).

Idea behind: Information messages are rebuild from the minimum possible of sets of blocks: Achieving min-cut capacity. Thus, high diversity is obtained.

Multiple user networks (cont'd)

- Challenges of DNCs: Network topology is varying but we want deterministic network codes.

Transfer matrix T of [Koetter & Medard]:

$$T = \begin{pmatrix} 1 & 0 & \cdot\cdot & 0 & \alpha_{1,1} & \cdot\cdot & \alpha_{1,M-1} & \cdots & \alpha_{1,M-1} \\ 0 & 1 & \cdot\cdot & 0 & \alpha_{2,1} & \cdot\cdot & \alpha_{2,M-1} & \cdots & \alpha_{2,M-1} \\ & & \cdot\cdot & & & & \cdots & & \\ 0 & 0 & \cdot\cdot & 1 & \alpha_{M,1} & \cdot\cdot & \alpha_{M,M-1} & \cdots & \alpha_{M,M-1} \end{pmatrix}$$

Channel outage means the corresponding variables as 0s.

We need to consider all error patterns.

Common coding solution for a dynamic topology.

Multiple user networks (cont'd)

- Performance analysis: Outage probability calculation.

Thorem 1: The diversity order for M-user cooperative networks with DNCs is 2M-1.

Proof outline: The worst case for a user is when all inter-user channels in outage. It means the error probability shall be lower if any partner can decode the messages. In the worst, it has a probability of P_e^{2M-1}.

Multiple user networks (cont'd)

- Simplified DNCs (MDS code based): Any M columns of T are linearly independent, (only consider one outage pattern)

- No significant performance degrading

  Theorem 2: Simplifed DNCs can achieve the diversity order 2M-1.

  Proof outline: Probabilities analysis...

Conclusions of Part I

- Proposed a new way of combining linear NCs and cooperative communications. The codes are designed such that the source can be rebuilt from a minimum possible set of network coding symbols (min-cut in dynamic topology of cooperative networks).
- We show diversity order is 2M-1, higher than previous schemes M. To simplify code construction, we propose DNCs based on MDS codes, which also have diversity order 2M-1 and much easier to construct.
- The proposed scheme can substantially improve the energy efficiency.

Part II: Multi-user multi-relay networks

Based on:

M. Xiao, J. Kliewer and M. Skoglund, "Design of network codes for multiple-user multiple relay wireless networks," *IEEE Transactions on Wireless Communications. Revision submitted in August 2010.*

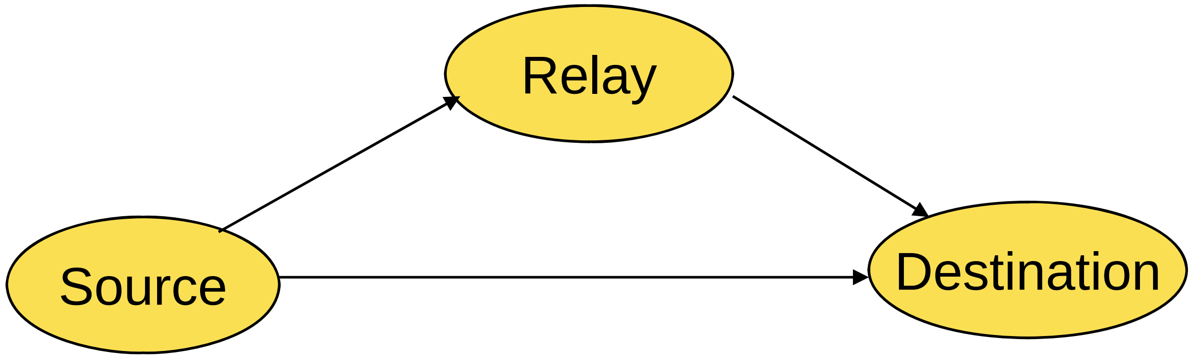
Background

- Compared to cooperative communications, separate relay has advantages:

(1) Easier to adapt to existed systems.

(2) More flexible in the network topology.

(3) Better performance is also expected.



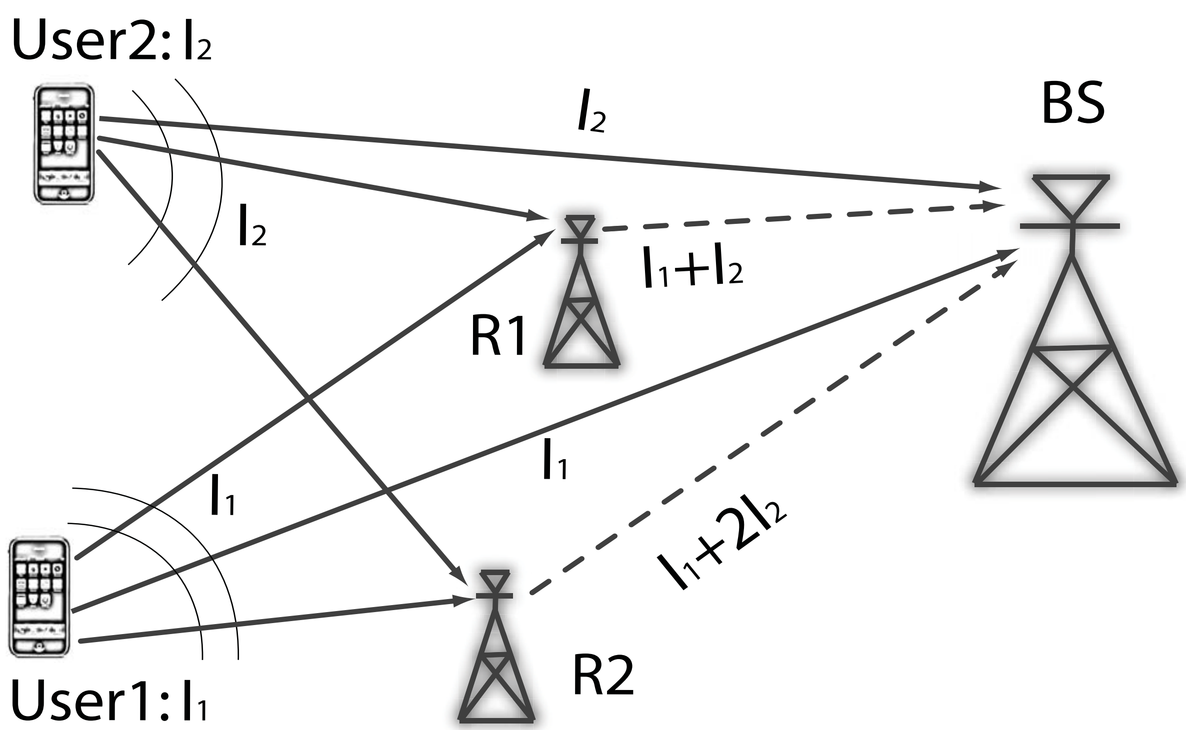
- Classic three node relaying: One-source one-relay. For multi-user networks, resource sharing in relaying (e.g., TDMA) degrades efficiency.

Background (cont'd)

- With increasing data-rate for multi-user networks, cellular networks use more and more relaying nodes: Networks are becoming denser and smaller.
- It will be common that multiple users simultaneously access two or more relays.

Two-user two-relays networks
--System description

- Channel models: Slow fading, Quasi-static, A common model in LTE. Orthogonal channels.
- Relay protocol: Selective decoding and forward (NC is on top of channel codes). Full diversity 3.



We want both user achiving diversity order 3.

MDS codes: I_1, I_2, I_1+I_2, I_1+2I_2 in GF(4). Any 2 of 4 can rebuild sources I_1 and I_2.

M-user N-relays networks

- For more general multi-user multi-relay networks, the design is more challenging.
- Similar to cooperative networks, MDS-based network codes can achieve full diversity N+1.

$$\mathcal{K} = \begin{pmatrix} 1 & 0 & \cdot\cdot & 0 & \gamma_{1,1} & \gamma_{1,2} & \cdot\cdot & \gamma_{1,N} \\ 0 & 1 & \cdot\cdot & 0 & \gamma_{2,1} & \gamma_{2,2} & \cdot\cdot & \gamma_{2,N} \\ & & \cdot\cdot & & & \cdots & & \\ 0 & 0 & \cdot\cdot & 1 & \gamma_{M,1} & \gamma_{M,2} & \cdot\cdot & \gamma_{M,N} \end{pmatrix}$$

- For transfer matrix, any M columns are indepent. Cauchy matrix, Vandermonde matrix works.

M-user N-relays networks

- We show that the diversity order is N+1.

Theorem 1, the diversity order based on MDS network codes is N+1 (full diversity).

--- Proof outline: If one S-R channel is in outage, we just remove the corresponding column. However, this occurs in probability.

M-user N-relays networks

- Another interesting coding scheme is SC (another way of combing information in relaying).
- SC can be regarded as a soft/analog network coding.

We still use decode and forward approach. In a relay i, Z_i messages can be decoded. The network codeword is thus

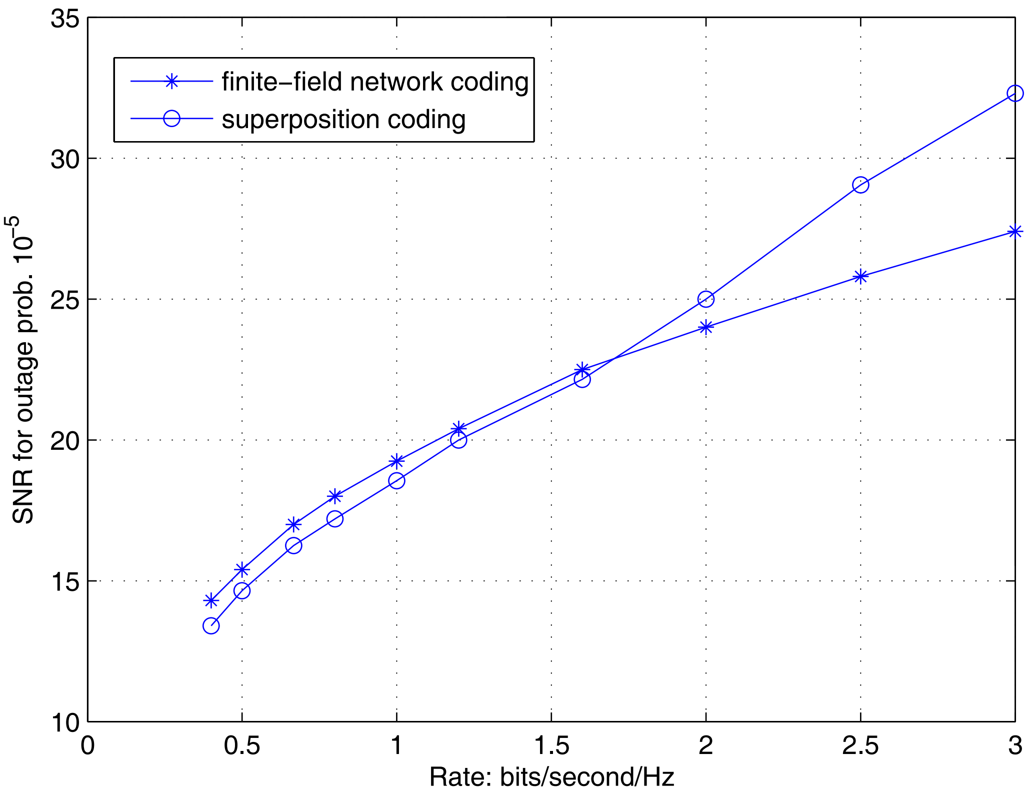
$$X_{r_i,0} = \frac{1}{\sqrt{Z_{r_i}}}(X_{1,r_i} + X_{2,r_i} + \cdots + X_{Z_{r_i},r_i})$$

In the BS, MMSE-SIC receiver is used. MI of user_j is

$$\mathrm{MI}_j = \frac{1}{M}\log_2\Bigg(1 + |a_{j,0}|^2\mathrm{SNR} + \frac{\frac{1}{Z_{r_1}}|a_{r_1,0}|^2 P_s}{N_0 + \frac{Z_{r_1}-1}{Z_{r_1}}|a_{r_1,0}|^2 P_s} \cdots + \frac{\frac{1}{Z_{r_K}}|a_{r_K,0}|^2 P_s}{N_0 + \frac{Z_{r_K}-1}{Z_{r_K}}|a_{r_K,0}|^2 P_s}\Bigg).$$

M-user N-relays networks

- Numerical results for 2-user 2-relay networks (SC is obtained by simulations)



•In high rate region, finite field NC is beter; In low rate, SC is better.

M-user N-relays networks

- More formally:

Proposition 1: For sufficient large SNR, MUMR relaying schemes based on FFNC have larger outage capacity than schemes based on SC schemes (MMSE-SIC decoding).

Proposition 2: For network without direct channels, the diversity order (with proposed codes) is N-M+1 ---- Singleton bounds

Conclusions of Part II

- We study the design of efficient NC for multi-user multi-relay networks.
- We show the MDS-code based NC is optimal in term of diversity order.
- We compared SC (soft NC) and network codes (finite field NC). Show that FFNC is better than SC in high rate (or high SNR) regions.

---- All above based on orthogonal channels, can we do better with non-orthogonal ones (MAC)?

Part III: Diversity-multiplex tradeoff analysis

Based on:

C. Wang, M. Xiao, and M. Skoglund, “Diversity-multiplexing tradeoff analysis of coded multi-user relay networks,” IEEE Transactions on Communications, (Submitted August, 2010)

Motivations

The problem of full-diversity (reliability) is addressed as above.

More questions

(1) How about rates? --- If we want to measure rates and diversity simultaneously.

(2) Is the relaying protocol the best? We drop all source messages at a relay if any one message cannot be decoded.

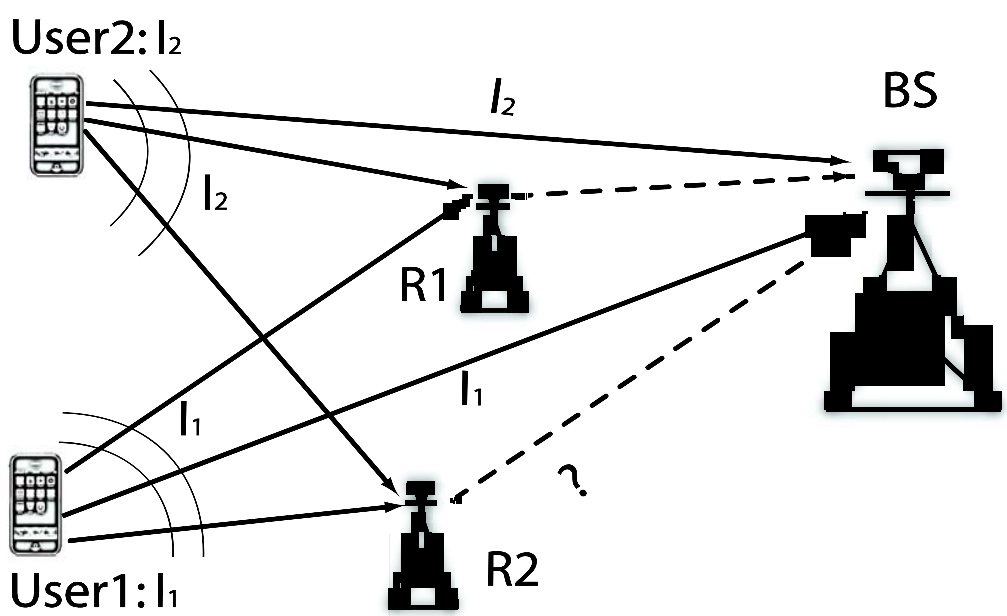
(3) Also the access protocols, we assumed orthogonal protocols. How much do we lose, compared to MAC?

Network model

Diversity-multiply-tradeoff (DMT) can simultaneously measure rates and diversity for high SNR.

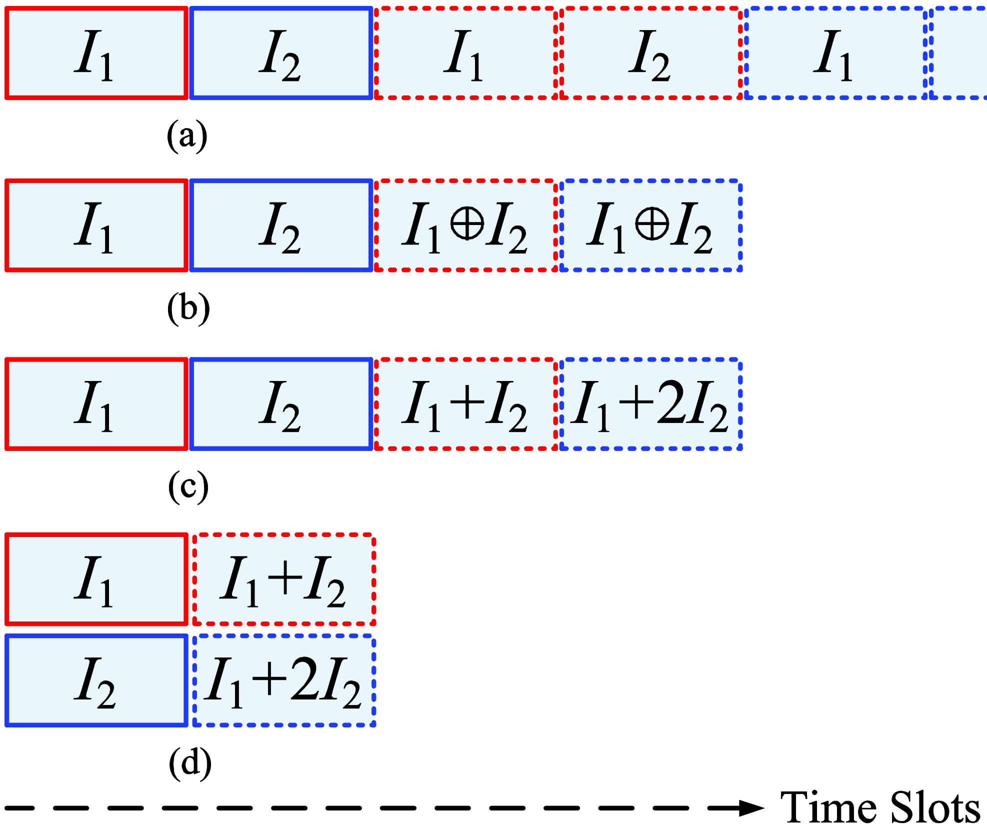
$$d = -\lim_{\rho\to\infty} \frac{\log P^o(\rho)}{\log \rho} \quad \text{and} \quad r = \lim_{\rho\to\infty} \frac{\bar{R}(\rho)}{\log \rho},$$

Reconsider two-user two-relay networks, slow fading



Transmission Protocol

Different transmission protocols:



(a) Standard protocol (b) BNC protocol (c) Orthogonal FFNC protocol (d) Non-orthogonal FFNC protocol.

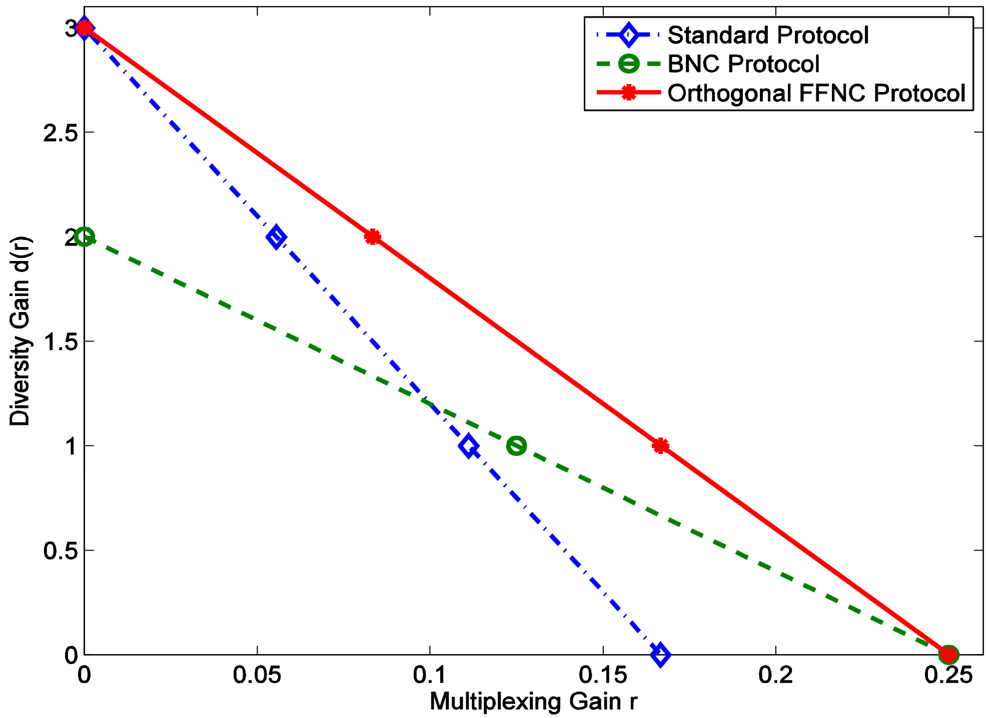
DMT analysis

Analysis results, we can show

(a) Standard protocol: $d = 3(1 - 6r)^+$

(b) BNC: $d = 2(1 - 4r)^+$

(c) Orthogonal FFNC: $d = 3(1 - 4r)^+$

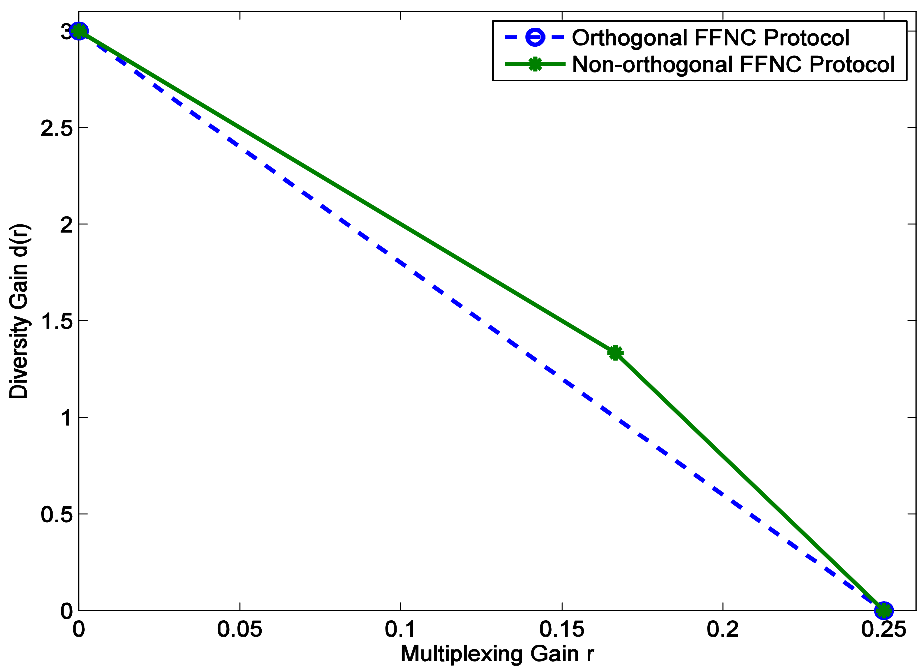


Our NC scheme strictly outperforms other schemes.

DMT analysis

Non-orthogonal FFNC MAC protocol.

$$d = \min\{(3-10r)^+, (4 - 16r)^+\}$$



MAC can improve DMT for this 2-U 2-R network. Orthogonal channels have lower complexity.

Is this always true?

General M-user K-relay networks

Theorem 1: The achievable DMT of networks with orthogonal channels and FFNC is

$$d = (K+1)(1 - (M+K)r)^+$$

Theorem 2: The achievable DMT of networks with non-orthogonal channels and FFNC is

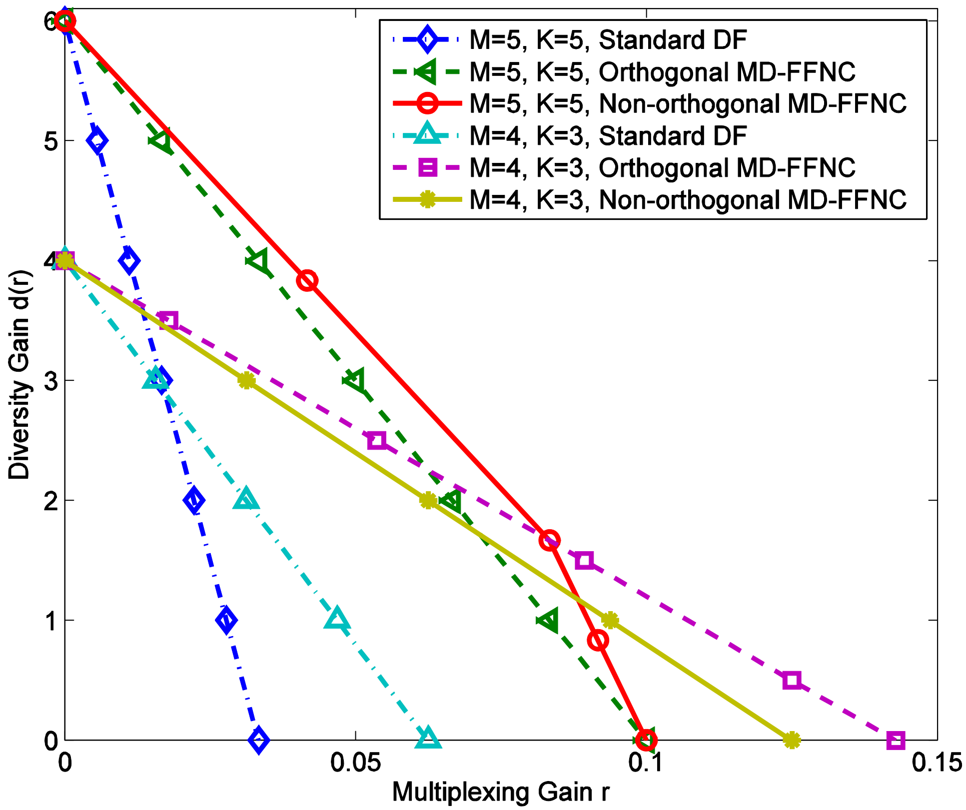
$$d = \min\{(K+1)(1 - 2(K+1)r)^+, M(1-2Mr)^+\}, M > K$$

$$d = \min\{(1-2r^+, M(1 - 2Mr)^+\} + K(1-2Kr)^+, M<=K$$

General M-user K-relay networks

(1) MAC is better for all r if K = M or K = M + 1.

(2) Orthogonal access is better in all r if K = M – 1 or M – 2.

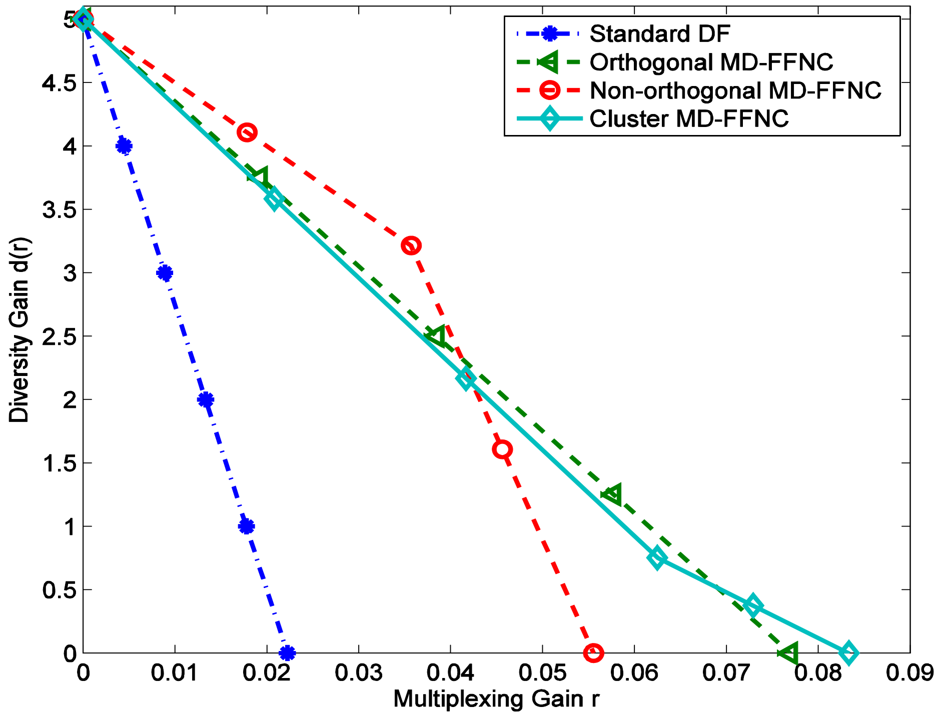


Different from one-hop communications, MAC does not necessarily outperform orthogonal channels.

General M-user K-relay networks

(3) K > M+1, MAC is better, if $\frac{K}{MK+M+K^2+K-2} \leq r \leq \frac{1}{2M}$

(4) K < M-2, MAC is better, if $0 \leq r \leq \frac{M-K-1}{2M^2-MK-K^2-M-K}$



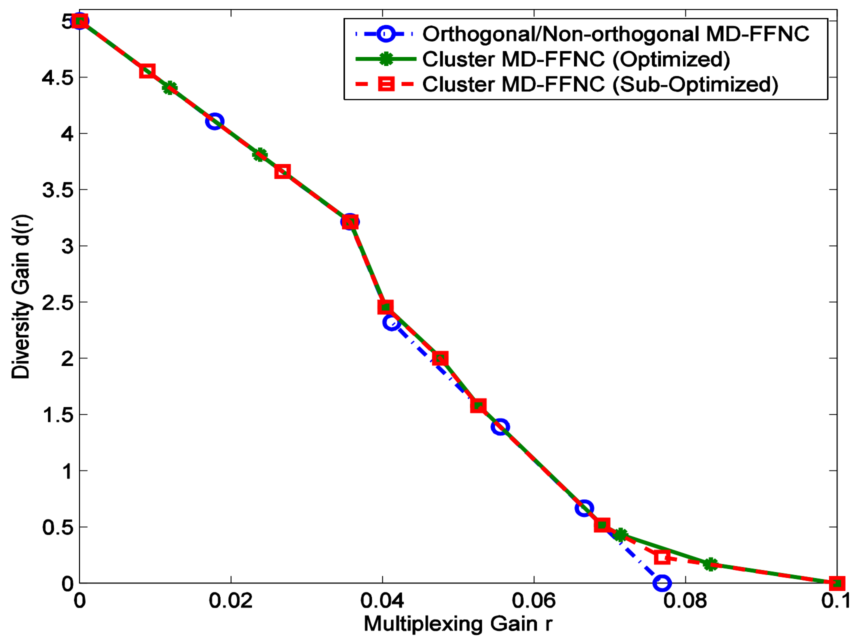
M = 9; K = 4;
The observations motivate us cluster (group) user and relay. Among clustering, orthogonal; Within cluster, MAC.

General M-user K-relay networks

New protocol: clustering FFNC protocols. MAC is for the nodes in the same cluster; Orthogonal for different cluster nodes. Different clustering strategy leads to different DMT performance.

An optimization problem is formulated: Cannot be solved in general.

Theorem 3: For given clustering number, DMT is maximized when the source is evenly divided among clusters.



Example M = 9; K = 4.

Note: for MAC, the reciever uses SIC decode Partially decoded messages from different MAC channels will not be dropped. They are useful for NC decoding. Individual outage probability are solved [4].

[4] R. Narasimhan, “Individual outage rate regions for fading multiple access channels,” in *IEEE International Symposium on Information Theory (ISIT), Nice, France, Jun. 2007.*

Conclusions of Part III

- Different from one-hop communications, MAC (non-orthogonal) may not be always better than orthogonal in DMT.
- Appropriate network coding scheme is very important for DMT performance.
- Appropriate clustering protocol and MA can improve DMT performance.

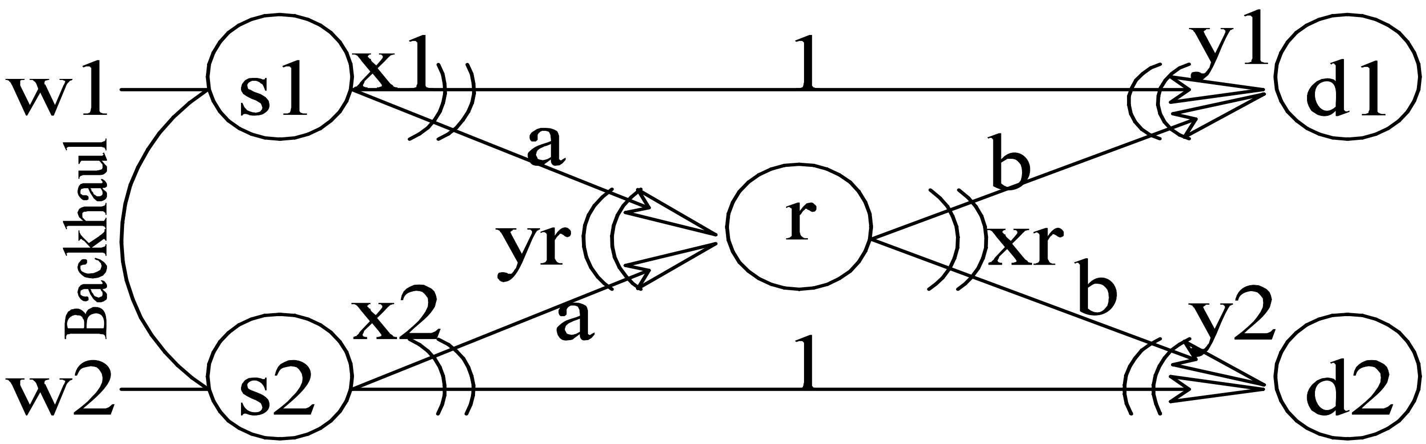
Part IV: Coding for Backhaul-supported Relaying Networks

Based on:

J. Du, M. Xiao, and M. Skoglund, "Cooperative Network Coding Strategies for Wireless Relay Network with Backhaul," IEEE Transactions on Communications, (Submitted August, 2010)

System Model

We consider a two-source multicast wireless network with one relay and backhaul supporting.



S1 and S2 have backhaul connection with capacity larger than sum rates.

r is a full-duplex relay. Gaussian channels.

Both messages W1, W2 are to be multicasted to d1 and d2.

There is a backhaul, compared to [1]

[1] D. Gunduz, O. Simeone, A. J. Goldsmith, H. V. Poor, and S. Shamai, “Multiple multicasts with the help of a relay,” http://arxiv.org/abs/0902.3178v1

System Model

Motivations: (1) Compared to wireless channels, connection between base-station is much faster.

(2) The network is a combination of relaying, MARC, BRC, source cooperation and network coding.

Symmetry channel gains (can be generalized):

$$\begin{aligned} Y_1^{(n)} &= X_1^{(n)} + bX_r^{(n)} + Z_1^{(n)}, \\ Y_2^{(n)} &= X_2^{(n)} + bX_r^{(n)} + Z_2^{(n)}, \\ Y_r^{(n)} &= aX_1^{(n)} + aX_2^{(n)} + Z_r^{(n)}, \end{aligned}$$

Differences from Gaussian MIMO relay: (1) Individual power constraint for each source; (2) Relay uses network coding instead of forwarding; (3) Can be extended to finite rate backhaul (on going work).

Cooperative Schemes

As relaying networks [2], the whole transmission has B blocks.

Successive decoding, backward decoding, SW-decoding are used

(1) DF + Finite field network coding

Relay uses ML decoding to decode both messages of previous block and network coding (XOR). Source also obtain network codewords by backhaul.

$$
\begin{aligned}
Y_{1,b}^{(n)} &= \sqrt{\alpha_1 P_1} V_1^{(n)} + (\sqrt{(1-\alpha_1)P_1} + b\sqrt{P_r}) U^{(n)} + Z_1^{(n)}, \\
Y_{2,b}^{(n)} &= \sqrt{\alpha_2 P_2} V_2^{(n)} + (\sqrt{(1-\alpha_2)P_2} + b\sqrt{P_r}) U^{(n)} + Z_2^{(n)}, \\
Y_{r,b}^{(n)} &= a\sqrt{\alpha_1 P_1} V_1^{(n)} + a\sqrt{\alpha_2 P_2} V_2^{(n)} + a(\sqrt{(1-\alpha_1)P_1} + \sqrt{(1-\alpha_2)P_2}) U^{(n)} + Z_r^{(n)}
\end{aligned}
$$

U is for network codewords (coherently add at the receiver), V1 is for the next w1, V2 for the next w2.

[2] T. M. Cover and A. El Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inf. Theory, vol. 25, pp. 572–584, Sep. 1979.*

Cooperative Schemes

(2) DF + superposition coding at relay (*linear* NC)

Relay addes two messages at signal domain.

(3) Lattice coding at relay

Relay does not decode source separately but decodes a function of two messages (Also called Compute-Forward).

(4) Network Beam-forming, S1, S2, trans X = f(W1, W2)

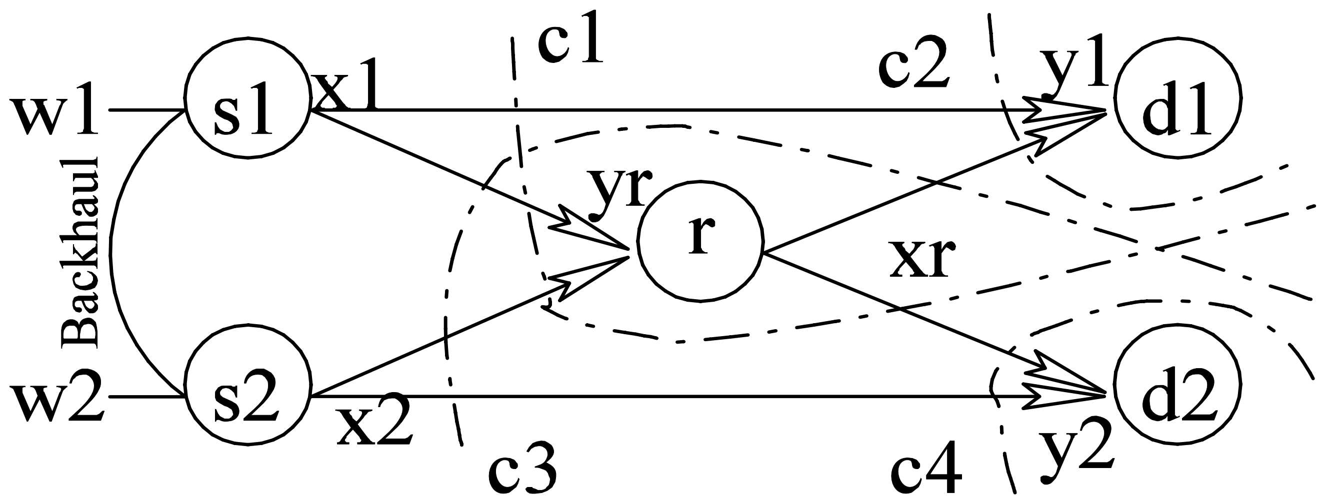
Network coding at S1, S2 --- to fully exploit coherent combination. Source messages are first exchanged by backhaul b-1, then the function of both messages are transmitted at time b at source, relay transmit at time b+1.

- We derive the achievable rate regions of all these schemes.

Cut-set bounds

Maximum-flow (maximum achievable sum-rate) is upper bounded by the min-cut (MI between two sides of a cut).

For our networks



$$R_1 + R_2 \le C_{cut-set} = \sup_{p(X_1,X_2,X_r)} \min\left\{\frac{1}{n}I(X_1,X_2;Y_1,Y_r|X_r), \frac{1}{n}I(X_1,X_r;Y_1), \frac{1}{n}I(X_1,X_2;Y_2,Y_r|X_r), \frac{1}{n}I(X_2,X_r;Y_2)\right\}$$

Cut-set bounds

We obtain the closed-form expression of cut-set bounds. The equation for general networks is too long. For symmetric networks R1= R2 = R, P1 = P2 = P,

$$R < \sup_{0\leq\alpha,\rho\leq 1} \min\left\{\frac{1}{2}C\left(P\left(1+b^2+2b\sqrt{1-\alpha}\right)\right),\right.$$
$$\left.\frac{1}{2}C\left(P[(1+2a^2)\alpha+2a^2\rho\alpha+a^2(1-\rho^2)\alpha^2 P]\right)\right\}$$

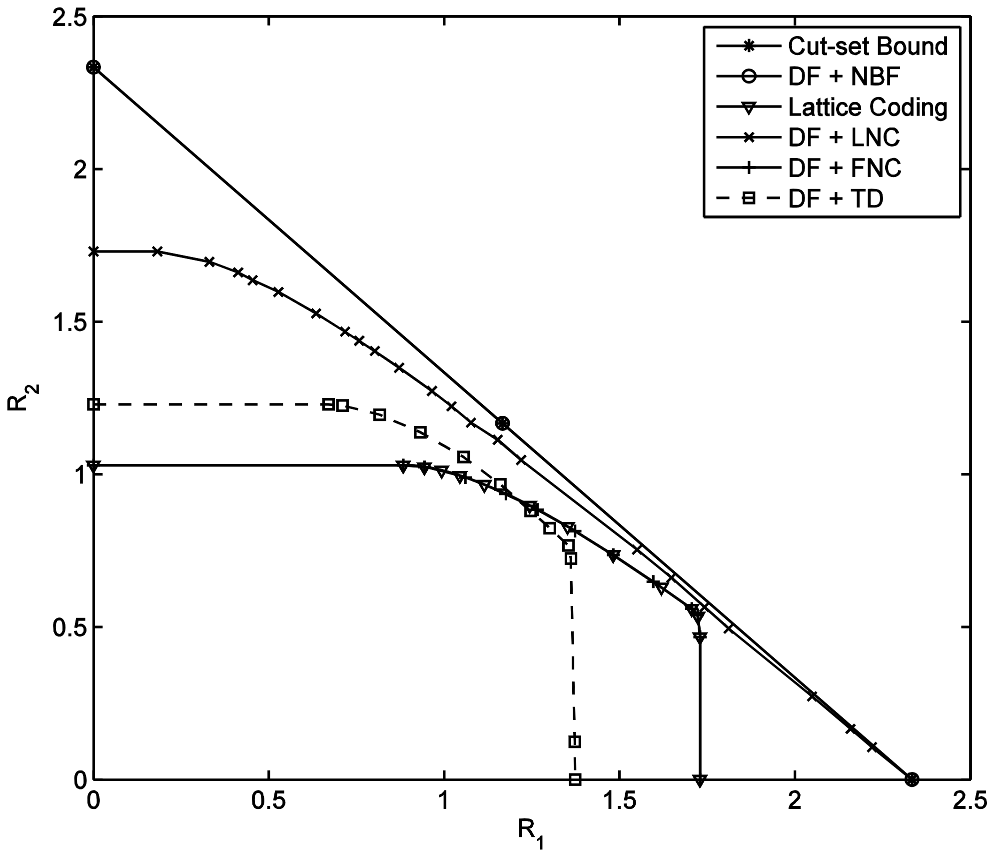
For the proposed NBF, we have

Proposition: In the symmetric scenario, NBF achieves the cut-set bounds, if and only if

$$\begin{cases} a^2 > \max\{1/2,\ (1+b^2)/4\} \\ 0 < P \leq \frac{8a^2(2a^2-1)}{2a^2(1+b^2)-b^2+\sqrt{(4a^2-b^2)(4a^2-1)b^2}}. \end{cases}$$

Numerical Results

Achievable rate regions



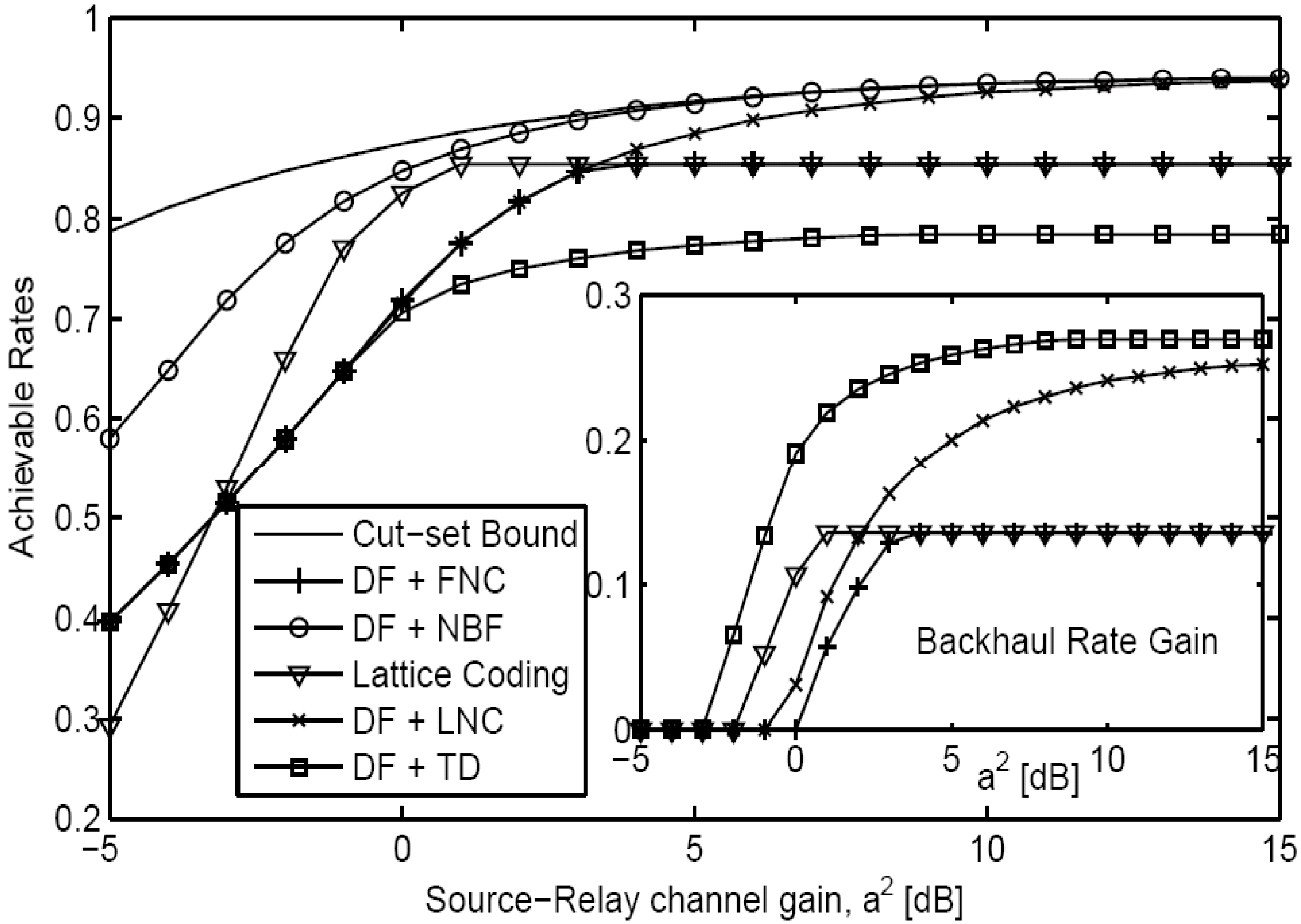
P_1/N_0 = 10 dB;

P_2/N_0 = 5 dB;

P_r/N_0 = 5 dB

Numerical Results

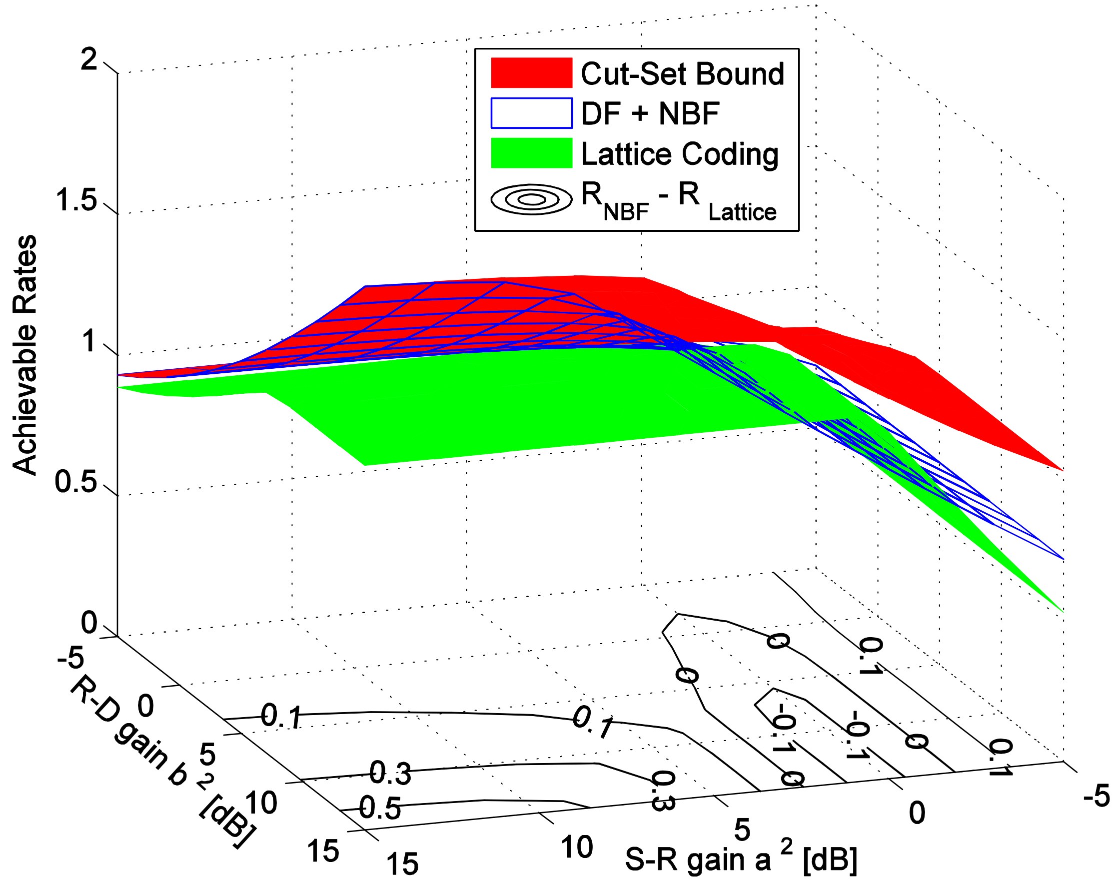
Impact of source-to-relay gains (a^2) to achievable rate regions, S-D gain (b^2) is fixed to 0 dB.



Comparison of Schemes with vs. Without backhaul is in subfigure.

Numerical Results

Comparison of NBF, cut-set bound and Lattice codes



Conclusions

(1) Backhaul can lead to significant rate.

(2) Cut-set bounds are studied and we show it can be achieved in certain condition by the proposed NBF.

(3) In most rate region, the NBF has the highest rates.

(4) Compared to TD without NC, NC can substantially increase rates.

Manuscripts can be downloaded at:

http://www.ee.kth.se/~mingx